Admission examination

Lomonosov Moscow State University

«Geometry and quantum fields»

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1. Find the following limit:
$$\lim_{t\to 0}\frac{\ln\cos 2t}{t^2}.$$

2. Evaluate the following indefinite integral:
$$\int x^5 \sin x^3\, dx.$$

3. Find the equilibrium points of the system
$$\begin{cases}\dot{x} = y - x - x^2\\ \dot{y} = 3x - y - x^2\end{cases}$$
For each point, determine whether the corresponding solution is stable or unstable.

4. Given a quantum harmonic oscillator with Hamiltonian $\frac{1}{2}(p^2+\omega^2 q^2)$, where q and p are the coordinate and momentum operators, respectively, find the expectation value of the operator q^4 in the ground state.

5. A particle of mass m and electric charge e moves through a uniform magnetic field $\overrightarrow{B}$? Determine the motion of the particle in the form $x_i = x_i(t)$, $i = 1,2,3$ in some suitable Cartesian coordinate system.

6. Consider independent eigenvectors of length 1 of the matrix
$$\begin{pmatrix}1 & -2 & -2\\ 1 & 4 & -2\\ 1 & 5 & -3\end{pmatrix}.$$
Find the volume of the parallelepiped formed by these eigenvectors .

7. Classify up to isomorphism abelian groups of order 324.

8. Determine the energy spectrum of a quantum system whose classical limit is described by the following Lagrangian:
$$L(x,y,\dot{x},\dot{y}) = \frac{1}{2}(\dot{x}^2+\dot{y}^2) - \frac{\alpha^2}{2}(5x^2 - 2xy + 5y^2)\,.$$