Entrance examination
for applicants to the Master's degree program
of Lomonosov Moscow State University
'Geometry and Quantum Fields'
in the area of Mathematics
2022

2022-07-29

1. Find the area of the figure on the coordinate plane located in the region $x > 0$, $y > 0$ and bounded by the lines $x^2+y^2 = 4$, $x^2+y^2 = 9$, $y = x$, $y = \sqrt{3}x$.

2. A die (hexahedral) is rolled one or more times until the sum of the rolled points is greater than 3. Find the probability that this will require exactly 2 rolls.

3. Solve the differential equation $xy' - y = x^3e^x$.

4. Solve for the positive integers x, y, z the system of equations

$$\begin{cases} x + y + z = 20, \\ x + 5y + 8z = 100. \end{cases}$$

5. Points $A(1, 2, -2)$, $B(3, 1, 4)$, $C(2, 0, 3)$, $D(-1, 2, 1)$ are given in space. Find the angle between planes ABC and ABD.

6. Solve the matrix equation

$$\begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix} X = \begin{pmatrix} 1 & 5 & 7 \\ 4 & 5 & 4 \end{pmatrix}.$$

7. Find all singular points of the complex variable function

$$f(z) = \frac{1}{z(z-i)^2}$$

and their types, and at each of these points calculate the residue.

8. Study for absolute and conditional convergence the series

$$\sum_{n=1}^{\infty} \frac{\cos n \cdot \arccos \frac{1}{n}}{\sqrt{n}}.$$