Entrance examination
for applicants to the Master's degree program
of Lomonosov Moscow State University
'Geomerty and Quantum Fields'
in the area of Mathematics
2022

2022-07-26

1. Find the maximum value of the function $z = x^2 - y^2$ on the line $y = 2x - 3$.

2. Solve the equation $40x + y = 5xy$ for the integers x, y.

3. The probability distribution of the random variable ξ is defined as

$$p_\xi(x) = \max\left(0, \frac{1}{2} - \frac{1}{4}|x-3|\right).$$

Find $\mathrm{P}(\xi \leqslant 4)$.

4. Solve the equation $z^3 + iz^2 + 2i = 0$ for the complex variable z.

5. Find the area of the figure on the plane aOb consisting of all points (a, b) satisfying the condition

$$\max_{x\in[0;1]} |ax + b| \leqslant 1.$$

6. For every $a > 0$ study the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^{n-1}}{(an^2 + n + 1)^{(n+1)/2}}.$$

7. Calculate the integral

$$\int_0^1 dx \int_x^1 dy \int_y^1 e^{z^3}\, dz.$$

8. Find the equilibrium points of the system

$$\begin{cases} \dot{x} = y - x - x^2 \\ \dot{y} = 3x - y - x^2 \end{cases}$$

For each point, determine whether the corresponding solution is stable or unstable.